CHREV 204

THEORY OF MULTICOMPONENT CHROMATOGRAPHY

A STATE-OF-THE-ART REPORT*

FRIEDRICH G. HELFFERICH

Department of Chemical Engineering, The Pennsylvania State University, University Park, PA 16802 (U.S.A.)

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1 INTRODUCTION

No three inventions have had greater impact on the working day of the physical scientist of our time than the computer, the photocopier and the chromatograph. Of these, at least one, the chromatograph, can rightly be called a Russian invention, attributable to Tswett[1], who, in 1906, described his success in separating chlorophyll components on a calcium carbonate column. As has often been the case with a fundamentally new idea, an incubation period was needed before the full potential of chromatography was recognized and realized, a development that owes much to the genius of Nobel laureate A. J. P. Martin[2]. Today, the chemist, biochemist, chemical engineer, geologist, metallurgist, nuclear engineer, clinical laboratory technician, almost anyone in practical physical science would find it hard to imagine how he could conduct his work without the almost instantaneous information on composition of matter that chromatography can supply.

The principal impact of chromatography has been as a fast, convenient and reliable tool of chemical analysis. The vast majority of publications and applications to date relate to analytical separations, to the extent that thinking about chromatographic problems has been conditioned to follow along the trails the pioneers of analysis have blazed This has not always been so. Indeed, Tswett seems to have had a preparative rather than analytical separation in mind, and much of the early prac-

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tical and theoretical work on chromatography, predating the gas chromatography revolution of the 1950s, had this orientation And in the present decade, preparative chromatography seems to be moving again out of the shadows cast by the immense success of analytical chromatography.

It will serve us well not to lose sight of the fundamental difference in objectives between analytical and preparative separations. Chemical analysis calls for a separation as complete as reasonably possible, in a single operation; small sample size and low concentration are advantages in that they reduce the amount of material required (and expended), and have been made possible by the marvels of detector technology of today Much of the work on chromatography —and some most spectacular progress— has been in reduction of analysis time, sample size and equipment size, without sacrifice in sharpness of separation. In contrast, a preparative separation calls primarily for processing of large quantities of material and recovery at concentrations as high as possible; sharpness of separation can be compromised to some extent as cuts with overlaps of components can easily be recycled. The difference in goals and outlook cuts deeper than is apparent at first glance, as an examination of the theoretical basis of chromatography can show.

From this vantage point, analytical chromatography presents itself as a science and art based on the fundamental equation for the average rate of travel, in direction of mobile phase flow, of a species i with partition coefficient K_i between the stationary and the mobile phase

$$v_i = v^0/(K_i + 1) \tag{1}$$

(for symbols see list at end of paper). In various guises, usually expressed in terms of retention time or adjusted retention time, this equation is the starting point for almost all developments in analytical chromatography. Stationary phases are chosen so that different substances have different partition coefficients, and thus travel through the column at different speeds, affording separation The work of the analytical chromatographer now is to fight the effects that lead to spreading and distortion of the "peaks" and so to obtain a sharp separation in as small as possible a column in as short as possible a time This has been the mainspring of technical advance in analytical chromatography, to mention capillary columns, thin-layer chromatography and high-performance liquid chromatography as only some of the more remarkable examples.

The chemist or engineer attempting to design a preparative separation faces different problems. The large amounts of matter to be separated force him to work with bands occupying even initially a substantial length of column, making separation harder than with minuscule analytical samples. The desire for effective recovery forces him to work with high concentrations; here, the partition coefficients are no longer constant, and this leads to "self-sharpening" or "non-sharpening" behavior of the traveling concentration variations in his column (formation of "shocks" and "rarefaction waves", in the physicist's language), complicating separations. Worst of all, at high concentrations the various species in his column will interfere with one another. the curvature of a sorption isotherm at moderate to high concentration reflects the fact that the behavior of a solute molecule is affected by the presence of others of its kind, and it would then be naive to assume it to remain unaffected by

the presence of molecules of other species. What is needed here is theory of "multi-component" chromatography, accounting for such interference. It is not possible to use ordinary chromatographic theory as the starting point and introduce interference as a refinement. Rather, multicomponent theory is fundamentally different in that it must deal with simultaneous rather than single differential equations and that interference can produce entirely new effects On the other hand, this higher complexity and the lesser need for absolute sharpness of separation in preparative applications have let theory retain to this day quite simple assumptions regarding perturbations (deviations from local phase equilibrium, non-ideal flow pattern, etc), a stage of development which theory of analytical chromatography has long outgrown.

As one might expect in view of the preeminence of analytical chromatography, there has been no lack of attempts to develop preparative separations by scale-up of successful analytical techniques. This approach amounts to fighting the facts inherent in analytical chromatography that high concentration is detrimental to ease of separation and that separation necessarily entails a dilution of the components with the eluent. From the theoretical point of view, the true challenge in preparative chromatography is not to fight the complexities of high concentrations and interference, but to make them work for better efficiency. Indeed, they make it possible to achieve concentration as well as separation in the same operation, though at the expense of having to reconstitute the column for renewed use.

2 ELUTION *VERSUS* DISPLACEMENT

In conventional, analytical operation with injection of a sample into a continuous stream of carrier gas or solvent, *the components of the original mixture travel* at different speeds in accordance with eqn. 1, increasing the distances from one another while their peaks flatten owing to perturbations (see Fig 1, left). The distances between adjacent peaks increase in proportion with the distance traveled while the peaks themselves spread in proportion with the square root of that distance. Accordingly, one can attain any desired sharpness of separation by making the column long enough. In the language of preparative chromatography this mode of operation is termed "elution development"

In contrast, for preparative separation, the use of an agent with greater affinity
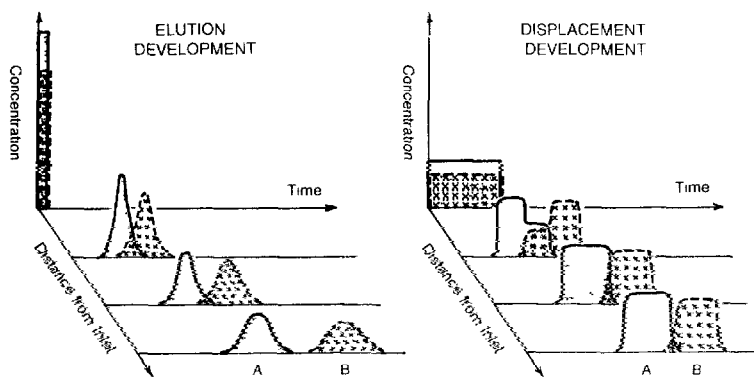

Fig 1 Comparison of elution and displacement development

for the sorbent than possessed by any component of the original mixture offers an advantage. Such an agent effectively displaces the mixture from the sorbent in a piston-type fashion. As the mixture is pushed through the column, its components arrange themselves in the order of increasing affinity for the sorbent. A state is attained in which bands of individual components follow one another close-up, with some overlap, all traveling at the same speed, and with a concentration essentially dictated by that of the displacing agent (see Fig. 1, right). Large sample size is an advantage rather than disadvantage in that the individual bands will be wider, so that overlaps constitute a lesser fraction of total material. If the agent is used at a sufficiently high concentration, the constitutents of the mixture can be recovered in concentrations much higher than charged The benefits for preparative separations are obvious. The price to be paid is that the column after separation is saturated with the strongly held displacing agent, which must be removed before another separation can be conducted. This oldest and most common mode of preparative chromatography is termed "displacement development".

Displacement development is by no means a recent innovation. The idea, first suggested by Tiselius[3] in 1943, predates the wide-spread adoption of gas chromatography It was used little later with spectacular success in the preparative separation of rare-earth elements in kilogram quantities and spectroscopic purity by Spedding and Powell[4] Indeed, one might speculate that preparative methods based on this principle would have become standard long ago and the philosophy of solving preparative problems by scale-up of analytical techniques might never have become prevalent, had not the phenomenal success of analytical gas chromatography focused attention on the elution development mode

3. EARLY THEORY IN RETROSPECTIVE

Chromatography in any form relies on differences in equilibrium sorption behavior and, conversely, much of the interest in sorption phenomena has been stimulated by the attempt to devise chromatographic and other separations. In the early days, before the gas chromatography revolution, preparative problems commanded much attention, and so theoretical developments included high-concentration and multicomponent behavior almost as a matter of course. Langmuir[5] derived his adsorption isotherm from the outset for multicomponent systems with interference, and the single-component form we have become used to associate with his name is but a later simplification Likewise, much of the early theoretical work on chromatography, most notably by DeVault[6] and Glueckauf[7-9], included non-linear isotherms and interference, making it applicable to high concentrations and multicomponent systems.

Specifically for displacement development, Claesson[10] in 1946 presented procedures and formulas to predict quantitatively the pattern of individual bands that would eventually arise and remain unchanged on further travel through the column. This left unanswered the practical question how much time and column length would be required to attain that final pattern. The way to solve this problem in principle by calculation of transient behavior was shown by Glueckauf[7], although it took much longer to develop mathematics capable of handling mixtures of more than three or four components without prohibitive calculation effort[11-14].

Apart from its great practical importance, displacement development is of *special interest in that it sheds an interesting sidelight on the evolution of multicomponent theory.* As will be discussed later, the conventional theoretical approach to chromatography runs into trouble when applied to the general case of arbitrary, gradual or repeated variations in feed composition. Displacement development involves introduction of two different feeds in succession: the mixture, followed by the displacing agent. Nevertheless, Glueckauf[7], with genius and intuition, correctly evaluated this situation, if with an assumption that at the time lacked a rigorous proof of validity and that is not applicable in the general case. This remarkable achievement may have helped to obscure the fundamental difficulty which a rigorous multicomponent theory for arbitary and repeated composition variations was facing.

4 THE CRUX OF MULTICOMPONENT THEORY WAVE EQUATION AND PARTIAL DERIVATIVES

Eqn. 1, although a starting point for analytical chromatography, is of little use in multicomponent systems. It is still valid (with composition-dependent K_i) but describes the rate of travel of molecules of a species rather than, what is much more convenient to evaluate, that of a given concentration or concentration variation. Its place is taken by the "wave equation"

$$v_{C_i} \equiv (\partial z/\partial t)_{C_i} \equiv v^{0\prime}[1 + (\partial \bar{C}_i/\partial C_i)_z] \tag{2}$$

easily derived from a differential material balance[15] and giving the velocity of a concentration C_i, that is, the velocity at which a given (constant) value of C_i travels through the column in the direction of flow. The trouble is with the partial derivative $\partial \bar{C}_i/\partial C_i$. If species i is the only solute present, this quantity (then a total derivative) is well defined as the variation in sorbed with fluid-phase concentration, it simply is the slope of the isotherm. In a multicomponent system with interference, however, the sorbed concentration $\bar{C}_i$ depends not only on the fluid-phase concentration C_i of the same species, but on those of all other solutes as well. This alone necessitates solving simultaneous equations. What is worse, however, is that even for a given composition $C_1, C_2, \ldots, C_n$ in an n-component system the derivative remains indefinite, being dependent on the local gradients of all concentrations. This becomes apparent without an excursion into calculus if one considers that the isotherm of
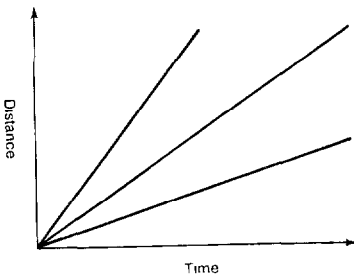

Fig 2 Trajectories of given concentrations C_i in distance-time plane (C_i constant along lines shown), traceable to single perturbation at column inlet and start of operation (schematic)

species i in the multicomponent system is a multidimensional hypersurface rather than a curve on a plane, and so allows a tangent to be placed at any specified point in an infinite number of directions in space, with different partial slopes $\partial \bar{C}_i / \partial C_i$.

For the simple case of constant feed and uniform initial column compositions it is plausible that one should be able to trace all concentration variations back to the same point in distance and time: the column inlet and time of first entry of feed (see Fig 2). The concentrations of all species coexisting at any point in space and time in the column should then have traveled jointly, from that same origin, and with the same speed for all. In other words, the wave equation should give the same velocity for all concentrations C_1, C_2, etc., that coexist at a distance–time point in the column, necessitating the partial derivatives to be equal for all species. This eminently plausible conclusion was intuitively accepted without proof by Glueckauf and his successors (with the notable exception of Baylé and Klinkenberg[16]), for this situation as well as for more complex cases with successive feed changes, such as displacement development Granted the equality of the partial derivatives, wave velocities can now be calculated.

Unfortunately, matters in reality are more complex. In the general case of arbitrary, variable starting conditions, the wave velocities of the species coexisting at a distance-time point in the column may well be unequal. What happens then is best understood in terms of a new concept, that of "coherence"

5 "COHERENCE"

Inherently, propagation of a perturbation in a system otherwise at equilibrium or steady state is a dynamic, transient phenomenon. It is over when the system has returned to its "stable" equilibrium or steady state. Yet, in a multicomponent system (more generally, in a system with several dependent variables), an arbitrary initial perturbation does not traval as such, but shakes itself down to a state it is more comfortable with. Such a state is called "coherent" (a word different from "stable" being needed)[12] Coherence thus is a state which a transient, non-equilibrium, non-steady state perturbation seeks to assume, in much the same way as a closed system
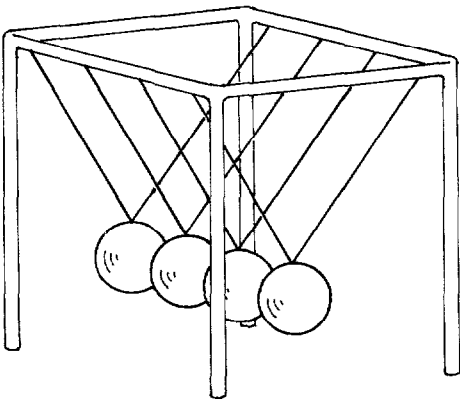

Fig 3 Array of suspended steel balls, for demonstration of attainment of a well defined dynamic state from arbitrary starting conditions

seeks to attain equilibrium and an open system with fixed boundaries and constant boundary values seeks to attain a steady state. This idea is unfamiliar to the physical chemist and chemical engineer, who have been preoccupied with equilibrium and steady state as conditions a system settles down to, it is not to the physicist and mechanical engineer, well acquainted with damped harmonic oscillations —say, the flutter of an aircraft wing

A manifestation of such attainment of a well defined transient state from arbitrary starting conditions can be given with a linear array of steel balls suspended by strings and just touching one another (see Fig. 3). An arbitrary initial variation soon settles down to a regular, fully synchronized motion in which the balls have stopped bouncing one another. (One might say they are wiser than mankind in that they understand how to achieve peaceful coexistence.)

The concept of coherence can be illustrated with a simple thought experiment[17]. Imagine a mixture of oil and water being sparged into a vertical tube initially filled with air. Granted that our oil is lighter than water, a layer of oil will form on top of the growing column of liquid in the tube and will increase in size as does that column (see Fig. 4). This experiment demonstrates that a single perturbation at the feed port (the switch from air to water–oil injection) is propagated through the tube not as a single wave, but as a set of waves traveling at different speeds. Here, there are two waves the air–oil interface and the boundary between the oil layer and the oil–water mixture In principle, any number of waves can arise, depending on the number of components of the system

Actually, the system in Fig. 4 can be said to involve three waves rather than only two, the third being the boundary between air and the oil–water mixture This wave, however, existed only for a fleeting instant, at the sparger at the start of oil–water injection, for buoyancy immediately saw to it that the oil layer would begin to form. In other words, that third wave was incapable of surviving and, instead, broke up to form the two waves that travel with integrity until they exit the tube at its upper end. In the language of coherence theory, the initial, non-surviving wave is
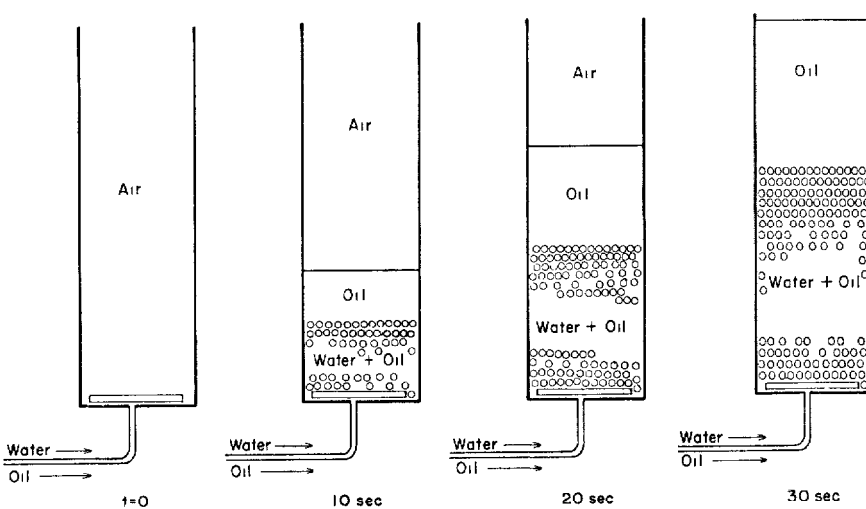

Fig 4 Water–oil injection into air-filled tube. shown at regular time intervals

called "non-coherent"; the two others, traveling with integrity, are called "coherent". In those words, the original non-coherent wave has been resolved into two coherent waves, which travel without further change in state.

From this simple model, a sufficient and necessary condition for a wave to be coherent is easily deduced. Any wave in a multicomponent system involves concentration variations of all species present at that point and can thus be viewed as a superposition of individual, single-component waves. if the wave is to be coherent, it must be definition travel with integrity rather than break up into several separate waves. Obviously, it can do so only if the wave velocities with respect to all component represented are equal. This is the fundamental coherence condition[18] (The presentation here, aiming only at conceptual understanding, glosses over the complications arising from the composition dependence of the coherent wave velocity as calculated from eqn. 2 and the coherence condition, which leads to formation of shock or rarefaction waves, for details, see ref. 12.)

The coherence condition, in combination with the wave equation, can now be seen to demand the equality of partial derivatives, the equality that was taken for granted by the early chromatography theoreticians This equality leads to an eigenvalue problem and allows only certain discrete composition variations rather than any arbitrary ones These variations can be mapped in a coordinate system with the species concentrations as axes (hodograph space) Such as grid of so-called composition paths[12] constitutes the groves into which the system will tend to settle. An example of a calculated grid is shown in Fig 5. Once the grid has been constructed, the response of the system to almost any perturbation is readily deduced without much further recourse to mathematics[12,17], relying on the principle that, given enough time, the system will settle down to entirely coherent behavior and thus follow a route exclusively along the paths of the grid. Observing a few simple selection rules one can trace that route much as one would project a vacation trip by car on a road map.

Of particular interest in this context is interference of waves as will occur when faster waves generated later will catch up with slower ones generated earlier (see Fig.
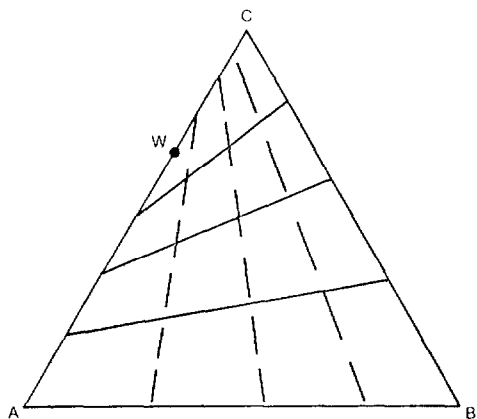

Fig 5 Composition path grid, calculated for three-component ion-exchange system A–B–C with constant separation factors $\alpha_{AB} = 2$ and $\alpha_{AC} = 5$
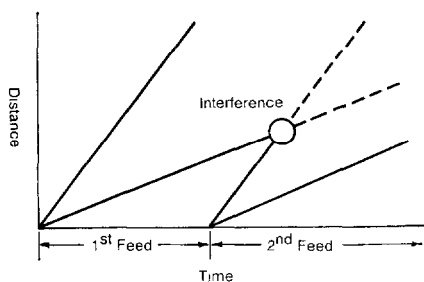
Fig 6 Interference of waves wave trajectories in distance–time plane for two stepwise feed composition variations (schematic)

6) In principle, such collision of two waves gives rise to local non-coherence. This non-coherence does not know or care whether it was introduced at the feed port at start or arose from wave interference later and somewhere in the column; it is unable to survive and is resolved into new coherent waves under the same rules as apply to any non-coherent feed variation With use of the composition path grid the resulting waves are easily predicted, at least qualitatively[12,17]. Overall, the evolving picture much resembles wave interference phenomena in optics and mechanics The simplest case is interference of two shock waves (see Fig. 7, left) Here, non-coherence remains confined to a single point in the distance–time plane This has made displacement development an exceptionally simple case to calculate, for all its waves are shocks. The most difficult situation arises when two diffuse waves interfere; here, non-coherence persists within a finite distance–time region (see Fig. 7, right) as, incidentally, it does in the case of a gradual non-coherent initial or feed composition variation.

The possibility of presence of finite regions of non-coherence brings home the fact that not all problems of multicomponent chromatography can be solved taking the coherence condition for granted. Rather, theory of coherence has served to elucidate what this condition signifies and when it does and does not apply, and so, paradoxically, has been of greatest value in situations involving non-coherence.

For coherence to be relied upon as a key concept, proof is needed that an arbitrary initial perturbation will indeed evolve into a coherent pattern. For the idealized case of so-called equilibrium theory —that is, for local equilibrium between
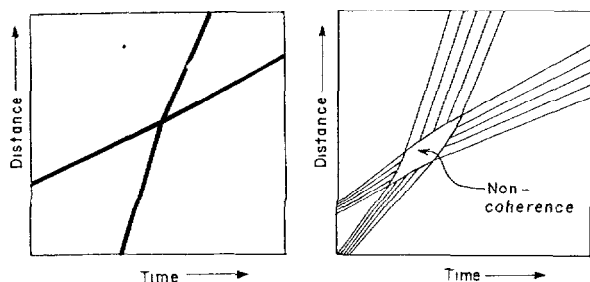

Fig 7 Types of wave interference interference of two shocks (left), with single point of non-coherence (at cross-over), of two diffuse waves (right), with finite distance–time region of non-coherence (schematic)

stationary and mobile phases, ideal plug flow, and absence of dispersion effects such as diffusion in axial direction— a rigorous proof based on the method of characteristics has been given[19] A brief outline of this proof is attached in the Appendix. In the language of the method of characteristics (see, for example, refs 20 and 21) the coherence principle can be formulated as follows.

An arbitrary starting variation, if embedded between sufficiently large regions of constant state, is resolved into simple (that is, coherent) waves between which new regions of different constant states arise.

It is indeed surprising that this relatively straightforward corollary of the method of characteristics seems not to have been explicitly formulated much earlier. The reason may be that the mathematician regards the statement as self-evident, but has in the past not realized its great predictive power.

It is interesting to note that the proof, based on the method of characteristics, is restricted to hyperbolic systems, as is that method, whereas the coherence principle appears to be more generally valid. Thus, if dispersion effects (axial diffusion, deviation from local phase equilibrium because of finite sorption-desorption rate, etc.) are included and contribute second-order terms to the material balance and wave equations, the method of characteristics is no longer applicable, but coherence is nevertheless approached asymptotically as the second-order terms fade out or dispersive and self-sharpening effects come into balance[22]. It should, therefore, be possible to find a more fundamental basis of the coherence principle, perhaps in statistics or statistical thermodynamics. No serious attempts in this direction have so far been made. Here is a challenge which, I hope, will eventually be picked up by experts in those fields

6 LANGMUIR-TYPE SYSTEMS, VACANCY GAS CHROMATOGRAPHY AND h TRANSFORMATION

The key to easy prediction of multicomponent column performance under almost any operating conditions is a knowledge of the composition path grid —that is, of the mapping of composition variations compatible with the coherence condition. Such grids can be constructed mathematically or graphically by solution of the eigenvalue problem that results from equating the wave velocities of the components. However, this procedure is cumbersome and time-consuming if the equilibrium equations are complex The art in working with the technique of composition path grids is to find short cuts how to construct at least approximate grids by calculating prominent features (such as loci of singularities, envelopes, etc.) and then filling in the rest of the grid by interpolation. In some instances of practical importance, however, even this can be dispensed with and simple algebraic relations can be obtained. Cases in point are sorption with multicomponent Langmuir isotherms and the mathematically equivalent situation of (stoichiometric) ion exchange with constant separation factors

The short cut applicable in these cases is provided by the so-called h transformation (see, for example, ref. 23). For ion-exchange chromatography as an example, the fluid-phase concentration variables (counterion equivalent fractions x_i) are transformed into new variables h_i obtained as the roots of the polynomial[23]

$$\sum_{i=1}^{n} x_i/(1 - \alpha_{1i}) = 0 \tag{3}$$

For Langmuir sorption systems a slightly different transformation[13,14] achieving the same purpose is algebraically more convenient. Velocities of coherent waves and compositions of zones between the waves can then be calculated from simple algebraic expressions involving the separation factors α_{ij} or coefficients of the Langmuir isotherm equations and the roots h_i of the initial and feed compositions. Even where non-coherence calls for numerical integration, the transformation offers substantial savings in effort in that the partial differential equations assume a much simpler form if written in terms of the h_i. Specifically, the velocity $(\partial z/\partial t)_{h_k}$ of a given root value h_k is a point property, that is, uniquely given by the local composition $h_1, h_2, ..., h_n$ rather than being dependent on gradients, as is the concentration velocity $(\partial z/\partial t)_{c_i}$.

In terms of the composition path grid —which now is superfluous— the transformation amounts to an orthogonalization[17,23] in the coordinate space of the transformed variables h_i, each set of paths is parallel to one axis and normal to all others. As a corollary, only one h_i varies across a coherent wave, all others remain constant.

In chromatography, the use of the h transformation (not under that name) was first suggested by Davidson[24] in 1949 for ion exchange in a discussion contribution that went largely unnoticed. The transformation has also been applied independently by Zhukhovitskii and co-workers[25,26] to a physically rather different situation in chromatography. These authors considered interference between components in gas chromatography at high concentrations through occupancy of space in the gas phase, while assuming constant partition coefficients. The most notable of his countless contributions in this context propose "vacancy chromatography"[25], where a pulse of inert gas is injected into a continuous gas stream of sorbable components, and "chromatography without carrier gas"[26], both with considerable interest for process control. Interestingly, even though composition-independent partition coefficients are assumed, the interaction of the components in the gas phase —loosely speaking, by taking away room from one another— again leads to mathematics for which Zhukhovitskii showed the h transformation to be applicable.

The h transformation, indeed, seems to be broadly applicable to systems with what one might call indiscriminate interference, that is, if species compete at constant relative strengths in sharing some function. This function may be occupancy of sorption sites in chromatography, occupancy of vapor space in vacancy chromatography or distillation, transport of electric current in electrophoresis, etc., and "indiscriminate" competition would be reflected in constant binary interaction coefficients: separation factors in chromatography, relative volatilities in distillation, relative electrochemical mobilities in electrophoresis.

The polynomial of eqn. 3 appears in differential geometry in the description of homofocal surfaces[27], and Binet is credited with having derived, in 1811, a theorem essentially equivalent to a reverse h transformation (see ref. 28). Over the years and in addition to the work previously quoted here, the transformation has been reinvented for multicomponent electrodiffusion by Pleijel[29] and Schlögl[30], multicomponent electrophoresis by Dole[31], and multicomponent distillation by Underwood[32] in his well-known stripping factor method, each author apparently being unaware of

previous uses. The broad applicability of the transformation to systems with several interacting dependent variables certainly deserves more attention, and it would be interestinging to determine the exact prerequisites for its use.

7. SELECTED RECENT APPLICATIONS

The most intriguing theory is of no more than academic interest unless it leads to progress in practical applications. Therefore, some such applications are mentioned here in passing. The selection is small, quite subjective, and confined to work inspired by or drawing heavily on theory; in particular, the selection also is unlikely to do justice to the many recent developments in this symposium's host country with which I am not sufficiently familiar.

Perhaps the most promising use that multicomponent chromatographic theory recently has found is in ongoing work on HPLC separations by displacement development[14] under Hováth, originator of HPLC (high-performance liquid chromatography). Molecules too large for efficient separation on conventional chromatographic sorbents can be analytically separated by HPLC. The combination of HPLC with displacement development extends the scope to preparative separations. For effective design, this work is drawing on multicomponent theory[12,13] and, incidentally, has provided experimental confirmation for a number of theoretical predictions

Another interesting development illustrating how multicomponent theory, suitably extended to more complex systems, can help to solve a difficult practical problem is the work by Wang and Huang[33] on removal of ammonium carbonate from physiological dialysis fluids. Here, it is essential that the pH and the concentrations of K^+, Ca^{2+}, and Mg^{2+} be maintained so that delicate subsequent readjustment becomes unnecessary. Aided by multicomponent theory, Wang and Huang achieved this with a mixed bed containing a zeolite and a weak-acid cation-exchange resin.

Other recent theory-inspired studies aim at improving the efficiency of preparative chromatographic operations by a program of injections alternatingly from one and the other end of the column (Bailly's and Tondeur's "two-way chromatography"[34]), by using temperature- or pressure-swing techniques (Camero and Sweed[35], Tondeur and Grevillot[36], and Wankat and Tondeur[37]), or by operating the column with moving feed ports (Wankat[38]).

An example of a fairly straightforward application of multicomponent theory (with h transformation) to an important practical problem in anion exchange is provided by the work by Clifford[39] on removal of nitrate from drinking water supplies also containing chloride, sulfate, and carbonate.

Work involving extensions of multicomponent theory has been carried out on systems with precipitation and dissolution of precipitates, namely, by Klein[40] with regard to scale formation in water treatment and by Walsh et al.[41] in connection with problems or uranium leach mining. Other work by Klein et al.[42] is concerned with carbonate regeneration in water treatment systems and incorporates ionic dissociation equilibria into multicomponent theory. In a similar application, work in our laboratory[43] predicted and experimentally confirmed pH excursions induced by association–dissociation reactions of weak-acid anions of buffers in multicomponent anion-exchange columns, demonstrating that in ion-exchange chromatography the pH is not necessarily what the analyst may think it to be

Ongoing work by Schweich *et al.*[44] entails extensions of multicomponent theory in a different direction by inclusion of catalytic reactions taking place on the chromatographic support.

In our laboratory, work on an extension of multicomponent theory to the general problem of propagation of perturbations in continuous counter-current mass-transfer operations such as fractionation, liquid–liquid extraction, gas absorption, and continuous counter-current ion exchange has been started by Hwang[45].

Still farther afield but addressing a clientele larger than the community of chromatographers are applications and extensions of multicomponent chromatographic theory to problems of enhanced oil recovery. These range from straightforward application to the cation balance in reservoir waters ion exchanging with clays in water or surfactant floods[46] to chromatographic behavior of surfactant mixtures[47,48] used in the latter and to an extension of multicomponent chromatographic theory to the presence of several fluid phases with arbitrary equilibrium partitioning of components and arbitrary flow properties[49,50], as may occur in surfactant, carbon dioxide, caustic, steam and other floods.

This brief catalogue, though far from comprehensive and biased by my own research interests, may serve to illustrate that multicomponent chromatographic theory is progressing, is finding many uses, and is stimulating advance in other fields. Undoubtedly, this symposium will add many other entries that would have deserved to be included.

8 SYMBOLS

C_i concentration of species i in fluid phase (mol m^{-3}, column*)
$\bar{C}_i$ concentration of species i in stationary phase (mol m^{-3} column*)
h variable in H function (eqn. 3)
h_i ith root of H function (eqn. 3) (transformed concentration variable)
K_i $= \bar{C}_i/C_i$ partition coefficient of species i
t time (s)
v_i velocity of molecules of species i
v_{C_i} velocity of concentration C_i
v^0 velocity of bulk fluid phase
x independent variable for coherence proof (Appendix)
x_i $= z_i C_i / \Sigma z_i C_i$ equivalent counterion fraction (z_i = electrochemical valence)
y independent variable for coherence proof (Appendix)
z distance from column inlet (m)
α_{ij} $= \bar{C}_i C_j / \bar{C}_j C_i$ separation factor of species i and j

9 APPENDIX COHERENCE PROOF[19]

A proof of attainment of coherence from arbitrary starting conditions can be briefly summarized as follows The proof uses the method of characteristics[20,21] and is phrased in the language of that method.

Stock-in-trade of the method of characteristics is that[20,21]

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* Note concentrations are per unit volume of column, not of respective phase

(a) In the plane of physical variables (distance and time in chromatography), regions traversed by characteristics originating exclusively from a region or regions of constant state are themselves regions of constant state.

(b) Regions traversed by characteristics of which one set originates from a starting variation, the other or others from a region or regions of constant state, are simple (*i.e.*, coherent) waves.

(c) Regions traversed by characteristics of which more than one set originates from a starting variation are non-simple (i e., non-coherent) waves.

In Fig. 8, for a system with two independent and two dependent variables, a starting condition is given along the curve AD in the physical plane, containing a variation along BC and constant states along AB and CD. The domain of dependence, BCF, of BC is a non-simple (non-coherent) wave as within it all characteristics have originated from the variation BC. Since the characteristics of the two sets have different slopes, all those originating from BC will cross one another inside BCF. The range of influence, FIH and beyond, of F thus is a region of constant state, with characteristics originating exclusively from the constant-rate starting regions AB and CD. Also, the regions BFHE and CGIF are simple (coherent) waves as each is traversed by one set of characteristics originating from the starting variation BC and another set originating from the constant-state starting region AB or CD.

This argument shows that the non-coherent starting variation BC is resolved into two coherent waves (BFHE and CGIF), between which a new region of constant state (FIH) arises.
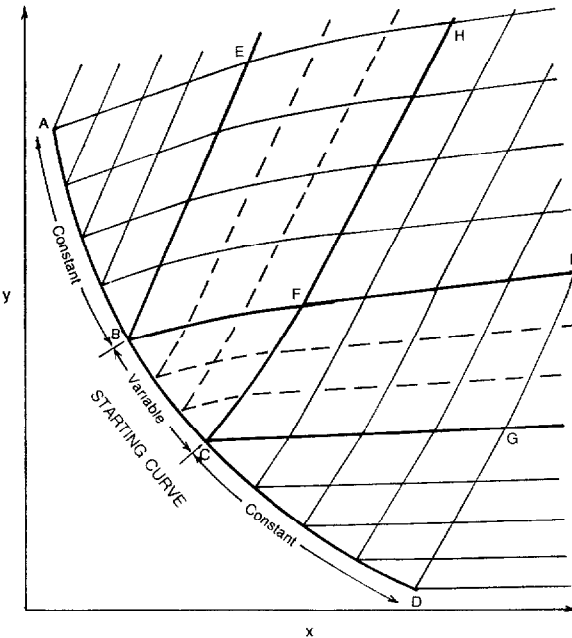

Fig 8 Physical plane with characteristics emanating from starting curve with regions of constant state AB and CD and variation BC, for system with two independent variables (*x* and *y*) and two dependent variables (two sets of characteristics) Characteristics originating from starting variation are shown as broken lines, those from regions of constant state as solid lines

The extension of the proof to more than two dependent variables is straightforward. The extension to more than two independent variables is much more difficult and has so far not been worked out in detail.

10 SUMMARY

Chromatography at high concentration and with large sample size, as desired for preparative separations, involves non-linear isotherms and interference of sorbable components with one another. Theoretical developments of the last fifteen years and their implications for separation methods are reviewed in the light of the concepts of wave propagation and coherence.

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